

Math 10A  
Quiz 9; Tuesday, 7/31/2018  
Time: 3 PM  
Instructor: Roy Zhao

Name: \_\_\_\_\_

Circle True or False. (1 point for correct answer, 0 if incorrect)

1. **TRUE** False The slope field of  $\frac{dy}{dt} = \sin(y)$  will be the same if we shift it left or right.

**Solution:** Since the differential equation does not depend on  $t$ , the slope is the same regardless what  $t$  is and hence is the same if we shift it left or right.

2. **TRUE** False It is impossible to have two unstable equilibria and no other equilibria.

**Solution:** If we think about this graphically, this is true, there must be at least a stable equilibria between those two unstable equilibria.

Show your work and justify your answers. Please circle or box your final answer.

3. (10 points) (a) (4 points) Use Euler's method to estimate  $y(3)$  with a step size of  $h = 1$  if  $\frac{dy}{dx} = 2x - y^2$  and  $y(1) = 1$ .

**Solution:** Repeatedly using Euler's method gives us

$$y(2) = y(1) + hf(1, y(1)) = 1 + 2 \cdot 1 - 1^2 = 1 + 2 - 1 = 2.$$

$$y(3) = y(2) + hf(2, y(2)) = 2 + 2 \cdot 2 - 2^2 = 2 + 4 - 4 = 2.$$

- (b) (5 points) Classify the equilibria of the differential equation  $P' = P(7 - P) - 10$ .

**Solution:** We expand and factor the right side as  $7P - P^2 - 10 = -(P - 2)(P - 5)$ . So, the equilibria are  $P = 2, 5$ . Drawing out the graph, we see that  $P = 2$  is unstable and  $P = 5$  is stable.

- (c) (1 point) What will  $P$  be in the future if initially  $P(0) = 3$ ?

**Solution:** Since  $\frac{dP}{dt} > 0$  when  $P = 3$ , we will go towards  $P = 5$ . So, the population will be 5 in the future.